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ORIGINAL ARTICLE

Transition from low to high dimensional chaos in a group of pulsations recorded in a broad radiowave interval

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Abstract We examined the dynamic characteristics of the time series regarding a group of pulsations in broadband spectrum at metric waveband solar radio emission. The data were recorded with the radio polarimeter of the INAF-Trieste Astronomical Observatory at July 17, 2002. The aim is to determine if the underlying process of these pulsations can be describe as a periodic, deterministic chaos or stochastic. The pulsations under inquiry in present paper are rather rare, as we found only one example of similar ones reported in the literature. Unlike most of the previously works where the analyses was done to a broadband pulsating events at one single frequency, we examine the pulsation event as it evolves both in time and in frequency. We found that the dynamics underlying the generation of pulsations can be characterized by a deterministic chaotic process which increases the dimension of chaos with frequency showing a transition from low-dimensional to high-dimensional deterministic chaotic system.

Keywords Chaos · Methods: data analysis · Sun: flares

1 Introduction

One of the unsolved fundamental problems in solar physics is the nature and evolution of trains of repetitive outlines of temporal fine structures which are highly structured in time. One of the approaches to understand the nature of such events is to find out if the underlying dynamics can be described as a regular, chaotic or stochastic process. For that, the basic assumption, following Takens' (1981) postulate, is that an observable time series (i.e. a sequence of observations, in our case, the solar flux) is the realization of some dynamical processes.

In particular, pulsation events are rather common in solar radio emission and they can be different as regards their frequency bandwidth, duration, etc. Pulsations seem to be the least complex phenomena among the other types of highly structured radio events showing, in general, the same structure in a rather large bandwidth. In Kurths and Herzel (1987a, 1987b) a metric-decimetric pulsation event lasting 40 s was analyzed.

In this paper we inquired the dynamic characteristics of the time series associated to a group of pulsations simultaneously recorded at 237, 327, 408, and 610 MHz where not only the polarization degree do not remain more or less constant with frequency (like a typical broadband pulsations event) but also the time structure changes. Unlike the previously mentioned papers, where the analyses was done to a broadband pulsating events at one single frequency, in this paper we examine the pulsation event as it evolves both in time and in frequency. The pulsations under inquiry in present paper are rather rare, as we found only one example of similar ones reported in the literature. Only Kurths et al. (1991) studied large band simultaneous pulsations at 16 different metric-decimetric radio frequencies (234, 336, 463, 610, among others) but in our event the considered data amount is larger and the pulsations at each frequency appear very homogeneous.

The main propose of this contribution is to determine if the underlying process of these pulsations can be de-

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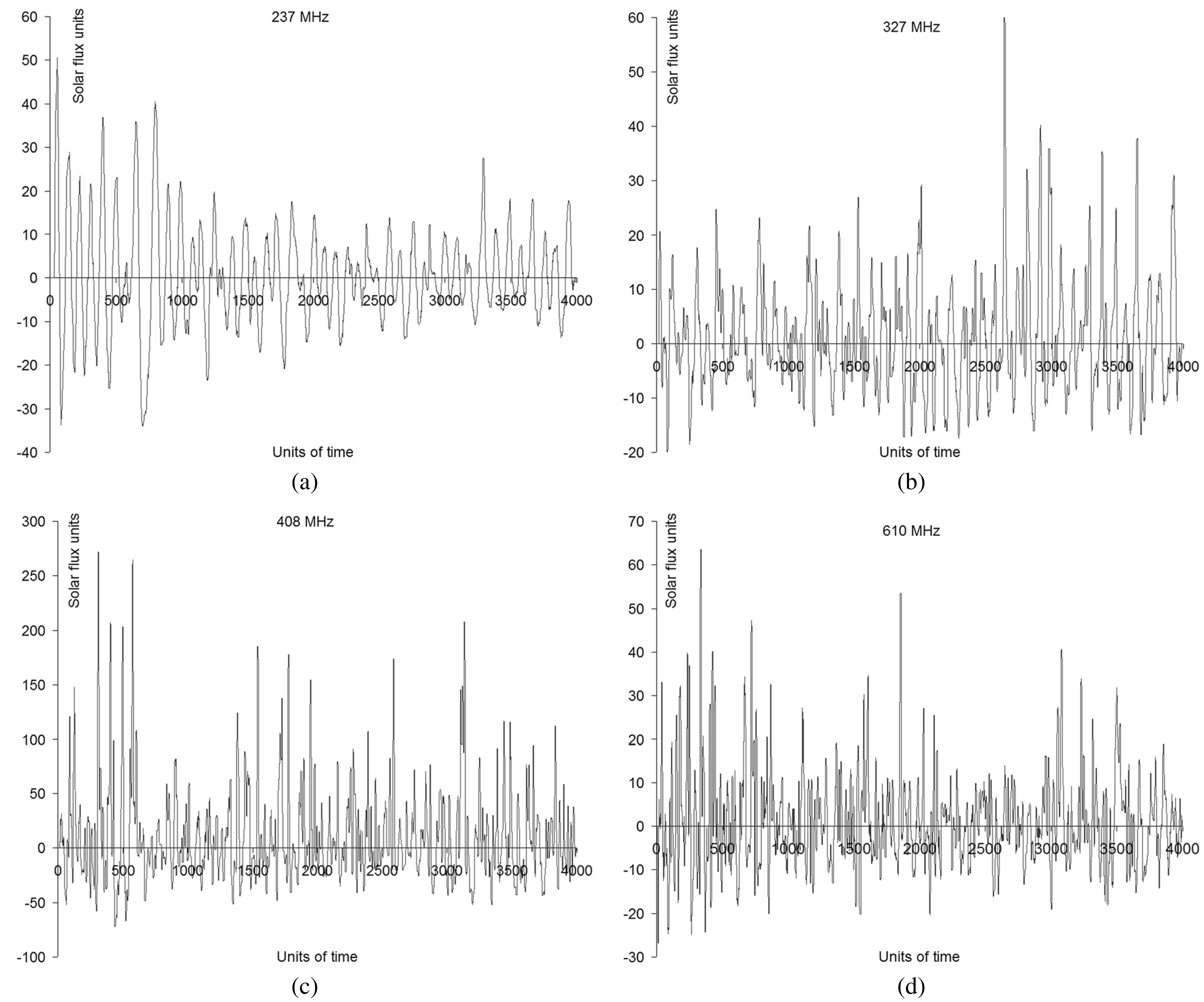

Fig. 1 Time profiles at 237 (**a**), 327 (**b**), 408 (**c**), and 610 MHz (**d**) of the pulsations (from 07:11:46 to 07:12:26 UT with sampling rate 100 Hz). The time intervals are the same

scribe as a periodic (or quasi-periodic), deterministic chaos or stochastic process. For that some temporal characteristics of associated time series are examined using some of the fundamental tools of the non-liner dynamical systems theory. In this sense an analogous approach was made for a series of fiber bursts in Méndez Berhondo et al. (2013).

2 Data

We used the July 17, 2002 data recorded by the radiopolarimeter of the Trieste Solar Radio System, INAF-Trieste Astronomical Observatory (Messerotti 2009 and Messerotti et al. 2003 a brief description of the instrument is included) with sampling rate 100 Hz (temporal resolution 10 ms). From 07:11:46 to 07:12:26 UT a group of pulsations recorded at 237, 327, 408 and 610 MHz was observed. Large bandwidth pulsations look rather similar but in this group of pulsations the profile are quiet different, so it is a rather exceptional one. The polarization was Right-handed. The associated optical flare was 1B (M8.5 in X-ray starting at 06:58 UT and end at 7:19 UT with maximum at 7:13 UT) located at N20W16 in the active region 10030 (NOAA classification). According to the Stanford magnetogram (Solar Geophysical Data) this position is inside southern magnetic field, so the emission was in ordinary magneto-ionic mode.

The calculations of parameters related to quantify the chaos are sensitive not only to the length and time resolution of time series but also to the noise. Noise in the original data was conveniently reduced using a median filter of 0.1 s. In order to normalize all pulsations referring the time series uniformly as pulsations around zero level, background oscillations were eliminated using a median filter of 2 s. Figure 1 shows the noise reduced and normalized time profiles

Table 1 Autocorrelation time (τ_{autocorr}) and number of structures for dimension estimation (N_s) (Isliker 1992) estimated for our data

Frequency (MHz)	τ_{autocorr} (data points)	N_s
237	20 (0.20 s)	200
327	13 (0.13 s)	308
408	08 (0.08 s)	500
610	07 (0.07 s)	571

of pulsations at 237, 327, 408, and 610 MHz. The selected data obey two fundamental criteria for its inquiry under non-linear dynamic systems theory: the length of the time series are sufficiently long and satisfy the reliable limit for the number of structures for dimension estimation (Isliker 1992):

$$N_s := N\Delta/\tau_{\mathrm{aucorr}} \geq 100 \tag{1}$$

where N_s is the number of structures (representing the number of full orbits in state-space), N is the length of the time series (in number of points), Δ is the temporal resolution and τ_{aucorr} is the autocorrelation time (time at which autocorrelation decays to $1/e$). See Table 1.

3 Results

These simultaneous pulsations do not appear as a broadband event in which duration and polarization degree remain approximately constant (Zlobec et al. 1987). In our case, the polarization degree of pulses increases with frequency from about 37 % at 237 MHz, about 70 % at 327 MHz and strongly polarized (80–100 %) at 408 and 610 MHz always in the right sense. Aschwanden (1986) reports strong and constant polarization when the flare position is not far from the solar disk center (0.50–0.56 units of solar radii) and lower and variable polarization in areas nearer to limb. Our result does not match with that as we find variable polarization in a rather central position (0.42 units of solar radii).

In consequence, we applied the time series analysis developed in the non-linear dynamic systems theory to right polarized component of the radio emission. Deterministic chaotic behavior can be detected from the inspection of observational time series representing real-world systems, revealing the nature of the temporal evolution of the underlying process. In the present paper we analyzed whether the observed group of simultaneously recorded pulsations at 237, 327, 408, and 610 MHz represent a stochastic process or are signatures of a deterministic chaotic behavior. For that, we estimated some fundamental parameters characterizing the nature of the temporal evolution of such events.

3.1 Determinism

Chaos is a deterministic however irregular, non periodic, process. Low dimensional chaos is a near-to periodic process and in the corresponding state-space an attractor is clearly noticeable. Contrary, high dimensional chaos is a more irregular process and must be proved that it is not really a stochastic process. One of the goals of the non-linear dynamical systems theory is to find out how deterministic the system is. Determinism is an important qualitatively key to decide if the dynamic of the system is generated by a deterministic, rather than a stochastic, process. In Kaplan and Glass (1992) a reliable determinism test (K-G determinism test) is introduced. The K-G determinism test is based on a proper reconstruction of the attractor of the system in the state-space. The state-space is reconstructed from the one-dimension scalar time series according an e-dimensional vector $\mathbf{x}(t)$ by the embedding method of time delay (Takens' theorem, Takens 1981):

$$\mathbf{x}(t) = \left[x(t), x(t+\tau), \ldots, x\big(t+(e-1)\tau\big)\right] \tag{2}$$

were t represents the time, e is the embedding dimension and τ is the time delay.

Takens' theorem asserts that the state-space of a dynamical system reconstructed according to (2) will have the same mathematical properties as the original system. The fundamental assumption underlying the Takens' idea is that an observable time series is the realization of some dynamical process.

For the state-space reconstruction of 237, 327, 408 and 610 MHz time series the optimal values of τ and e were estimated according to the first minimum of the average mutual information (Fraser and Swinney 1986) and the false nearest neighbor method (Kennel et al. 1992), respectively.

The state-space is then divided into equally sized boxes. As a deterministic system evolves the variables defining the system describe a set of trajectories in the state-space that are arbitrarily close together never departing from the region called attractor. These trajectories can be represented as vectors occupying each box, defining a vector field in the state-space.

The K-G determinism test measures the length of the vectors generated in the state-space occupying all boxes. If the system is fully deterministic, each resultant vector will be of unit length, and, consequently, the average length will be 1. On the contrary, for a stochastic system the average length tends to zero. Figure 2 shows the vectors field in the state-space for pulsations at 237 MHz after applying the K-G determinism test where the presence of an attractor is noticeable. Table 2a shows the calculated values of the averaged determinism factor (λ) for the pulsations considered in this paper. The number of boxes is high and we have vectors with different length (for example, 941 boxes with vectors sizes from 0,083 to 0,966 for 237 MHz). For this reason we have considerable values in uncertainty due to the large range in values for vector sizes. Table 2b shows the total number of boxes by frequency and number of boxes containing vector sizes longer than 0.8 and lower than 0.5.

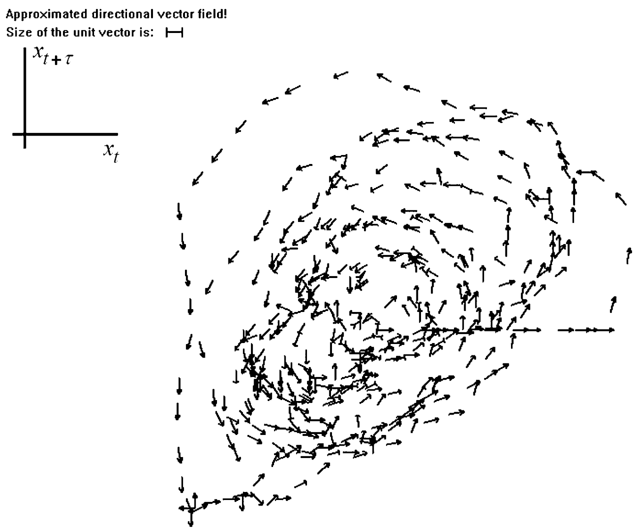

Fig. 2 Vectors field in the state-space for pulsations at 237 MHz after applying the K-G determinism test. The presence of attractor is noticeable

3.2 Pointwise dimension

Solar radio emission time series represent real-world systems that we expect to be nor fully deterministic. The previously applied determinism test can be considered as pointing out a decreasing of determinism factor with as the frequency where the pulsations are observed increases, that is, an indication for an increase of chaos dimension. However, this result should be tested with other calculations. In order to describe quantitatively the complexity of the dynamics of time series associated to pulsations at 237, 327, 408 and 610 MHz, we calculated the pointwise dimensions, D_P, (Farmer and Ott 1983) as a locally alternative variant of the correlation dimension $D^{(2)}$ to characterize the chaos regime based on the fact that local dimension estimations have the property that they can be used with non-stationary data. The way to calculate the pointwise dimension is calculating the probability p_i to find points in a neighborhood of a point χ_i with size r in the state-space (r is the radius of a state-space neighborhood around χ_i). The pointwise dimension is:

$$D_P(\chi_i) = \lim_{r\to 0}[\log p_i / \log r] \quad (3)$$

The calculated pointwise dimension is the average of all $D_P(\chi_i)$ over all points of the state-space. In Table 2a the calculated average of D_P for the group of pulsations are shown. The range for the D_P calculation was taken centered

Table 2a Determinism factor (λ), the average pointwise dimension (D_P) and the Hurst exponent (H) calculated for our data. Other well-known time series (natural and man-made) are included as comparison

Time series	λ	D_P	H
Pulsations at 237 MHz	0.789 ± 0.120	3.625 ± 0.597	0.68
Pulsations at 327 MHz	0.687 ± 0.160	5.319 ± 0.937	0.54
Pulsations at 408 MHz	0.547 ± 0.190	5.580 ± 1.151	0.36
Pulsations at 610 MHz	0.497 ± 0.194	6.303 ± 1.075	0.28
Lorenz model	0.984 ± 0.031	2.12[a]	0.76
Sunspots number	0.796 ± 0.107[2]	2.790 ± 0.462[b]	0.86[c]
White noise	0	7.806 ± 0.760	1.79×10^{-3}

[a]From Veronig et al. (2000)

[b]λ and D_p calculated for sunspots number from January 1749 to July 2014 (data from SILSO, World Data Center-Sunspot Number and Long-term Solar Observations, Royal Observatory of Belgium, on-line Sunspot Number catalogue: http://www.sidc.be/SILSO)

[c]From Kilcik et al. (2009)

Table 2b Total number of boxes by frequency and number of boxes containing vector sizes longer than 0.8 and lower than 0.5

Freq. (MHz)	No. boxes	No. boxes with $\lambda > 0.8$	% respect to all boxes	No. boxes with $\lambda < 0.5$	% respect to all boxes
237	941	284	30.2	101	10.7
327	1366	158	11.6	375	27.5
408	1410	40	2.8	953	67.6
610	1736	36	2.1	1410	81.2

on the optimal embedding dimension from the false nearest neighbor.

3.3 Hurst exponent

The Hurst exponent (H) is a dimensionless parameter introduced by Hurst (1951) as a statistical measure of the variability of a time series characterizing the self-similarity of a dynamic system. The used method was the most used and best-known method: the so called R/S or rescaled range method to estimate the Hurst exponent proposed by Mandelbrot and Wallis (1969) based on the previous work of Hurst (1951). The Hurst exponent is also commonly used as a measure of the geometric (fractal) scaling in the data series (Turcotte 1997). In this sense, the Hurst exponent measures the smoothness of a time series. Having values between 0 and 1, the Hurst exponent close to 0 means more jagged time series with high fractal dimension indicating an anti-persistent turbulent process characteristic of irregular time series. On the other hand, the Hurst exponent near 1 means a smooth time series having low fractal dimension typical for a persistent process with long term "memory". In Table 2a the values of Hurst exponent calculated for the studied pulsations are included.

4 Discussion

The values summarized in Table 2a for the determinism factor, the average pointwise dimension and the Hurst exponent show that the dynamic of the analyzed group of simultaneous pulsations in a broad radiowave interval cannot be described as a low-dimensional chaos. The determinism factor λ decrease as the frequency increases showing a not fully deterministic dynamical system. On the other hand, the dimensional analysis reveals a change in the chaos regime with frequency starting from a relatively low-dimensional chaos at 237 MHz and increasing up to the high-dimensional chaos at 610 MHz, according to the calculated values for the average pointwise dimension D_P. The correspondence between the decrease in the determinism factor λ and the increase in the value of pointwise dimension D_P that we find from 237 MHz to 610 MHz pulsations is well known. Finally, the Hurst exponent exhibits a similar behavior: starting with relatively high Hurst exponent at 237 MHz, it continuously decreases down to the rather low value at 610 MHz. The change of the Hurst exponent reveals the transit from a quasi-regular process with low fractal dimension to an irregular and jagged process with higher fractal dimension, which is in agreement with the calculated values for the dimension and determinism factor.

In Kurths and Herzel (1987a, 1987b) and Kurths et al. (1991) the dimensionality of pulsation events had been investigated revealing a low-dimensional deterministic chaos with dimension $D^{(2)} \leq 3.5$. In particular, Kurths and Karlický (1989) presented the dynamical evolution from a limit cycle to a low-dimensional deterministic chaos ($D^{(2)} = 2.0 \pm 0.2$) of broadband pulsations using a unique set of data. Contrary to us, all of these papers deal with broadband pulsation events considering one single frequency. Our results suggest that linear MHD oscillations model for pulsations (Robert et al. 1984) applied to one single frequency cannot explain the simultaneous group of pulsations in a broad radiowave interval, being more adequate non-linear equations for explains pulsations not locally but in a large scale.

In this sense, Farmer (1982) analyzed the nature of the chaotic attractor of a dynamical system described by a delay differential equation of the form

$$\dot{\mathbf{x}}(t) = F\big(x(t), x(t - t_d)\big) \tag{4}$$

where t_d is a time delay (do not confuse with the time delay τ in Eq. (2)). This differential equation describes systems in which a stimulus has a delayed response. As the delay t_d grows, an increase in the dimension of chaos is verified. Following this idea, we can explain our results qualitatively assuming a disturbance (source of the pulsations) being generated in a magnetic loop in a place higher than the 237 MHz source. Then the plasma waves (Langmuir, magnetoacoustic, i.e.) propagate reaching deeper layers (higher frequencies) with a delay. Some fundamental parameters of the plasma particle-wave dynamics (i.e. particles distribution, collisional damping rate, wave amplification growth rate, etc.) evolve non-linearly increasing significantly the dimension of chaos characterizing the system as the disturbance reaches deeper layers in the loop.

Dissipative cyclic self-organizing systems (wave-particle and wave-wave interactions) communicate with the environment are sensitive to changes of the ambient parameters—e.g. particles density (see Aschwanden 1987). Assuming ambient parameters strongly frequency-dependent we can expect in such dissipative cyclic self-organizing systems a transition regime to high-dimensional chaos. Being out of the focus of this contribution, how high-dimensional chaos arises as system parameters change is a study matter. In Harrison and Lai (1999) some characteristic features for the route to high-dimensional chaos are investigated from a theoretical approach.

The transition regime from the deterministic relatively low-dimensional to the high-dimensional chaos (as the increase of the D_P imply) follows a quiet smoothed evolution (Fig. 3), which is in conformity with Farmer (1982) where the transition to high-dimensional chaos in the system described by the delay differential equation of the form (4) is fairly smooth, and proceeds with a nearly linear increase in the number of positive Lyapunov exponents as t_d increases.

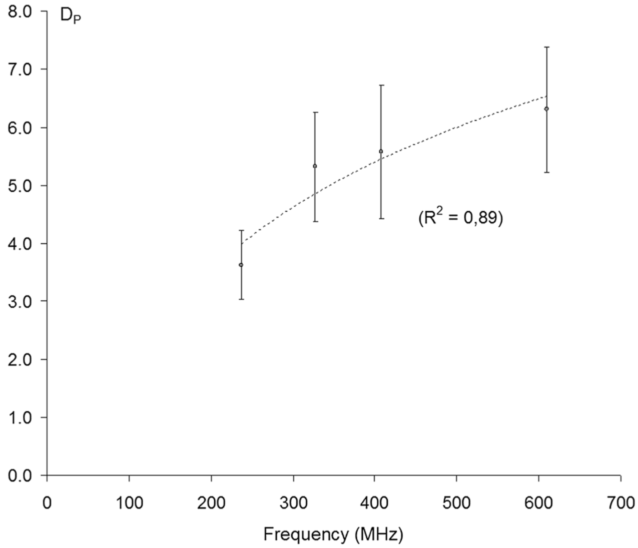

Fig. 3 Obtained values of pointwise dimension (D_P) for the time series associated to 237, 327, 408 and 610 MHz pulsation event. The transition regime from the deterministic relatively low-dimensional at 237 MHz to the high-dimensional chaos at 610 MHz follows a smoothed profile

5 Conclusions

In this paper we examined a pulsations event not only locally (at fixed frequency) but also in a large scale as a process evolving both in time and in frequency. The set of pulsations analyzed here are not common in the sense they can not be cataloged as a broadband pulsations. The dynamics underlying the generation of these pulsations can be characterized by a deterministic chaotic process which increases the dimension of chaos with frequency. That is, a non periodic behavior with an increasing of dependence on initial conditions increasing the complexity of the process generating the pulsations. Three main results can be summarized:

1. The pulsations simultaneously recorded at four metric wavelengths from higher level (237 MHz) to deeper ones (610 MHz) in the active region are not periodic showing a deterministic chaos behavior. In consequence, linear MHD oscillations model for broadband pulsations—like global sausage or kink type of free oscillations mode (i.e. Robert et al. 1984)—cannot explain this kind of simultaneous group of pulsations in a broad radiowave interval. Each group of pulsations at fixed frequencies can be described by a set of independent non-linear differential equations depending on the minimum number of freedom degrees fixed by the dimension of chaos in each case.
2. Furthermore, the dimension of chaos increases with frequency showing a transition regime from relatively low to high-dimensional deterministic chaos. As a global scale event the pulsations evolve following a non-linear model increasing the complexity of the process originating the pulsations with the frequency, being governed by a delay differential equation. The transition regime from the deterministic relatively low-dimensional to high-dimensional deterministic chaos could be qualitative explained through a system in which a stimulus has a delayed response. As the delay grows, an increase in the dimension of chaos is obtained.
3. Finally, pulsations are not governed by a purely stochastic process. However the number of variables which are necessary to describe the pulsations grows notably with frequency.

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